Getting ready for A Level

Maths

Week 5 – Solving Quadratics

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1. Solving by factorising
2. Solving by completing the square
3. Solving using the formula

Solving by factorising Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x^2 - 15x = 0$	
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Solving by factorising Examples

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$b = 7, ac = 12$	
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term $(7x)$ using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Solving by factorising Examples

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Solving by factorising Questions

1 Solve

a $6x^2 + 4x = 0$

b $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$

d $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$

f $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$

h $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$

j $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$

l $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$

b $x^2 - 3 = 2x$

c $x^2 + 5x = 24$

d $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$

f $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$

h $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Solving by factorising

Answers

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$ **j** $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

Solving by completing the square Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
$(x + 3)^2 - 9 + 4 = 0$	**2** Simplify.
$(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$	**3** Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	**5** Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**6** Write down both solutions.

Solving by completing the square Examples

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify.

$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	**6** Divide both sides by 2.
$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Solving by completing the square Questions

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$

b $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$

d $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$

f $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of the equation.

Solving by completing the square Answers

3 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$

b $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$

d $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$

f $x = \frac{-3 + \sqrt{89}}{10}$ or $x = \frac{-3 - \sqrt{89}}{10}$

4 **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$

b $x = \frac{-3 + \sqrt{23}}{2}$ or $x = \frac{-3 - \sqrt{23}}{2}$

c $x = \frac{5 + \sqrt{13}}{2}$ or $x = \frac{5 - \sqrt{13}}{2}$

Solving by using the formula Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Solving by using the formula Examples

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Solving by using the formula Questions

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Hint

Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Solving by using the formula Answers

5 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

6 $x = \frac{7 + \sqrt{41}}{2}$ or $x = \frac{7 - \sqrt{41}}{2}$

7 $x = \frac{-3 + \sqrt{89}}{20}$ or $x = \frac{-3 - \sqrt{89}}{20}$

8 **a** $x = \frac{7 + \sqrt{17}}{8}$ or $x = \frac{7 - \sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$

Further Maths GCSE Questions Part 1

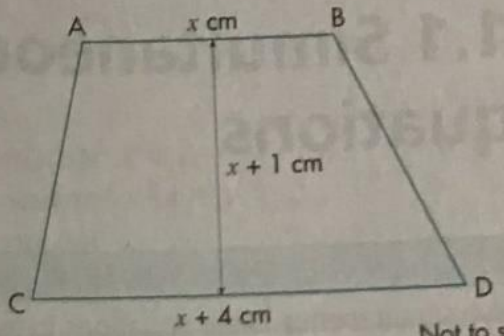
EXAM-STYLE QUESTIONS

1. ABCD is a trapezium. GRADE A

The area of the trapezium is 110 cm^2.

a) Set up an equation in x.

b) Solve the equation and hence find the length of CD.



Not to scale

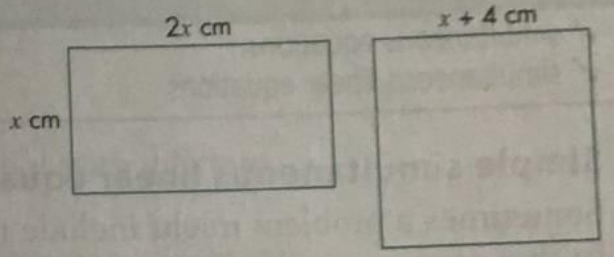
2. The diagrams show a rectangle and a square.

a) i) If the rectangle and the square have the same *perimeter*, use this fact to set up an equation in x.

ii) Solve your equation and hence find the length of the side of the square.

b) i) If the rectangle and the square have the same *area*, use this fact to set up an equation in x.

ii) Solve your equation and hence find the length of the side of the square in this case.



Not to scale

Further Maths GCSE questions part 2

GRADE A*

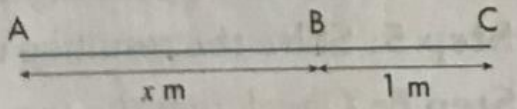
3. Find the values of a, b and c such that $6x^2 + 6x - 1 \equiv a(x+b)^2 + c$.

4. a) Find the values of a, b and c such that $3x^2 - 12x + 4 \equiv a(x+b)^2 + c$.

   b) Hence, or otherwise, solve the equation $3x^2 - 12x + 4 = 0$, giving your solutions correct to 2 decimal places.

5. ABC is a straight line.

   The length of BC is 1 m and the length of AB is x m.



Not to scale

   a) AB : BC = AC : AB

   Use this fact to set up an equation in x.

   b) Solve your equation and hence write the length of AB in metres in the form $a + b\sqrt{5}$.

Further Maths GCSE Questions Answers

Exam-style questions

1. a) $(x+1)(x+2)=110$

b) $x=9$ or -12 so the length of CD is 13 cm.

2. a) i $6x=4x+16$

ii $x=8$, the side of the square is 12 cm

b) i $2x^2=(x+4)^2$

ii $x=4\pm\sqrt{32}$, the side of the square is 13.7 cm

3. $a=6, b=0.5, c=-2.5$

4. a) $a=3, b=-2, c=-8$ **b)** $x=3.63$ or 0.37

5. a) $x=\dfrac{x+1}{x}$

b The solution is $x=\dfrac{1}{2}\pm\dfrac{1}{2}\sqrt{5}$ and the length of AB is $\dfrac{1}{2}+\dfrac{1}{2}\sqrt{5}$ m, since it can't be negative.