

Getting ready for A Level  
**Maths**

Week 6 – Simultaneous  
Equations

# Week 5 – Solving Quadratics

1. Solving linear simultaneous equations
2. Solving linear and non-linear simultaneous equations
3. Using graphs to solve simultaneous equations
4. Solving simultaneous equations with 3 unknowns

# Solving linear simultaneous equations

## Examples

Solve the following simultaneous (linear) equations.

$$2x + 3y = 3$$

$$4x - y = 13$$

$$x = 3, y = -1$$

### Method 1: Elimination

$$2x + 3y = 3$$

$$12x - 3y = 39$$

Adding two equations:

$$14x = 42$$

$$x = 3$$

Substituting back into second equation:

$$12 - y = 13 \rightarrow y = -1$$

### Method 2: Substitution

Rearranging second equation:

$$y = 4x - 13$$

Substituting into first equation:

$$2x + 3(4x - 13) = 3$$

$$2x + 12x - 39 = 3$$

$$14x = 42$$

...

This method will be mostly used for non linear equations.

You may need to multiply one or both equations to get equal coefficients.

# Solving linear simultaneous equations

## EXERCISE 11A

### GRADE B

1. Solve each pair of simultaneous equations.

a)  $x + y = 15$

$y = 2x$

d)  $y = x + 12$

$y = 3x$

b)  $x = 3y$

$x + y = 24$

e)  $y = x - 10$

$x = 5y$

c)  $x + y = 60$

$y = 4x$

f)  $x + 4 = y$

$y = 9x$

2. Solve each pair of simultaneous equations.

a)  $x + y = 20$

$x - y = 6$

d)  $y = 2x + 3$

$y = 8x$

b)  $y + x = 23$

$y - x = 5$

e)  $x + y = 20$

$y = 3x - 2$

c)  $x + y = 6$

$x - y = 14$

f)  $y = 2x + 4$

$y = 10 - x$

3. Solve these simultaneous equations.

a)  $4x + y = 17$

$2x + y = 9$

d)  $2x + 5y = 37$

$y = 11 - 2x$

b)  $5x + 2y = 13$

$x + 2y = 9$

e)  $4x - 3y = 7$

$x = 13 - 3y$

c)  $2x + y = 7$

$5x - y = 14$

f)  $4x - y = 17$

$x = 2 + y$

4. Solve these simultaneous equations.

a)  $5x + 2y = 4$

$4x - y = 11$

d)  $5x - 2y = 24$

$3x + y = 21$

b)  $4x + 3y = 37$

$2x + y = 17$

e)  $5x - 2y = 4$

$3x - 6y = 6$

c)  $3x + 5y = 15$

$x + 3y = 7$

f)  $3x - 2y = 3$

$5x + 6y = 12$

5. Solve these simultaneous equations.

# Solving linear simultaneous equations

**5. Solve these simultaneous equations.**

<b>a)</b> $2x + 5y = 15$ $3x - 2y = 13$	<b>b)</b> $2x + 3y = 30$ $5x + 7y = 71$	<b>c)</b> $2x - 3y = 15$ $5x + 7y = 52$
<b>d)</b> $2x + y = 4$ $x - y = 5$	<b>e)</b> $5x + 2y = 11$ $3x + 4y = 8$	<b>f)</b> $x - 2y = 4$ $3x - y = -3$
<b>g)</b> $3x - y = 5$ $x + 3y = -20$	<b>h)</b> $3x - 4y = 4.5$ $2x + 2y = 10$	<b>i)</b> $x - 5y = 15$ $3x - 7y = 17$

**6. Here are four equations.**  
A:  $5x + 2y = 1$     B:  $4x + y = 9$     C:  $3x - y = 5$     D:  $3x + 2y = 3$   
Here are four sets of  $(x, y)$  values.  
 $(1, -2), (-1, 3), (2, 1), (3, -3)$   
Match each pair of  $(x, y)$  values to a pair of equations.

**7. Find the area of the triangle enclosed by these three equations.**  
 $y - x = 2$      $x + y = 6$      $3x + y = 6$

**8. Find the area of the triangle enclosed by these three equations.**  
 $x - 2y = 6$      $x + 2y = 6$      $x + y = 3$

**SIMULTANEOUS EQUATIONS**

**HINTS AND TIPS**

Find the points of intersection of each pair of equations, plot the points on a grid and use any method to work out the area of the resulting triangle.

# Solving linear and non-linear simultaneous equations

## Examples - 1

$$\begin{aligned}x^2 + y^2 &= 17 \\x + 2y &= 2\end{aligned}$$

$$\begin{aligned}x &= 2 - 2y \\(2 - 2y)^2 + y^2 &= 17 \\4 - 8y + 4y^2 + y^2 &= 17 \\5y^2 - 8y - 13 &= 0 \\(5y - 13)(y + 1) &= 0 \\y &= \frac{13}{5} \text{ or } y = -1\end{aligned}$$

$$\begin{aligned}\text{If } y &= \frac{13}{5}, x = 2 - 2\left(\frac{13}{5}\right) = -\frac{16}{5} \\ \text{If } y &= -1, x = 2 - 2(-1) = 4\end{aligned}$$

**Step 1:** Rearrange linear equation to make  $x$  or  $y$  the subject.

**Step 2:** Substitute into quadratic equation and solve.

Common Schoolboy Error: To forget the  $+y^2$  that was already there.

**Step 3:** Use an equation (e.g from Step 1) to find the values of the other variable.

# Solving linear and non-linear simultaneous equations

## Examples - 2

A

$$y = x^2 - 3x + 4$$
$$y = x + 1$$

Note that no 'Step 1' is needed here because  $y$  is already the subject of the linear equation.

$$x + 1 = x^2 - 3x + 4$$

$$x^2 - 4x + 3 = 0$$

$$(x - 1)(x - 3) = 0$$

$$x = 1, x = 3$$

$$\begin{array}{cc} \downarrow & \downarrow \\ y = 2, & y = 4 \end{array}$$

I personally like using arrows because it makes clear which value of  $y$  corresponds to which  $x$ .

B

Solve the following, giving your solutions correct to 3 significant figures.

$$x^2 + y^2 = 7$$

$$2x + y = 1$$

**Bro Tip:** "Correct to 3 significant figures" suggests we won't have a nice solution, and hence we'll have to use the quadratic formula.

$$y = 1 - 2x$$

$$x^2 + (1 - 2x)^2 = 7$$

$$x^2 + 1 - 4x + 4x^2 = 7$$

$$5x^2 - 4x - 6 = 0$$

$$a = 5, b = -4, c = -6$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - (4 \times 5 \times -6)}}{10}$$

$$x = -0.766 \text{ or } x = 1.57$$

$$\begin{array}{cc} \downarrow & \downarrow \\ y = 2.53 & \text{or } y = -2.13 \end{array}$$



# Solving linear and non- linear simultaneous equations Questions

## EXERCISE 11B

### GRADE B

1. Use the substitution method to solve each pair of linear simultaneous equations.

a)  $2x + y = 9$   
 $x - 2y = 7$

b)  $3x - 2y = 10$   
 $4x + y = 17$

c)  $x - 2y = 10$   
 $2x + 3y = 13$

### GRADE A

2. Solve each pair of simultaneous equations.

a)  $xy = 2$   
 $y = x + 1$

b)  $xy = -4$   
 $2y = x + 6$

3. Solve each pair of simultaneous equations.

a)  $x^2 + y^2 = 25$   
 $x + y = 7$

b)  $x^2 + y^2 = 9$   
 $y = x + 3$

c)  $x^2 + y^2 = 13$   
 $5y + x = 13$

4. Solve each pair of simultaneous equations.

a)  $y = x^2 + 2x - 3$   
 $y = 2x + 1$

b)  $y = x^2 - 2x - 5$   
 $y = x - 1$

c)  $y = x^2 - 2x$   
 $y = 2x - 3$

5. Solve these pairs of simultaneous equations.

a)  $y = x^2 + 3x - 3$  and  $y = x$

b)  $x^2 + y^2 = 13$  and  $x + y = 1$

c)  $x^2 + y^2 = 5$  and  $y = x + 1$

d)  $y = x^2 - 3x + 1$  and  $y = 2x - 5$

e)  $y = x^2 - 3$  and  $y = x + 3$

f)  $y = x^2 - 3x - 2$  and  $y = 2x - 6$

g)  $x^2 + y^2 = 41$  and  $y = x + 1$



# Solving simultaneous equations graphically

## Example

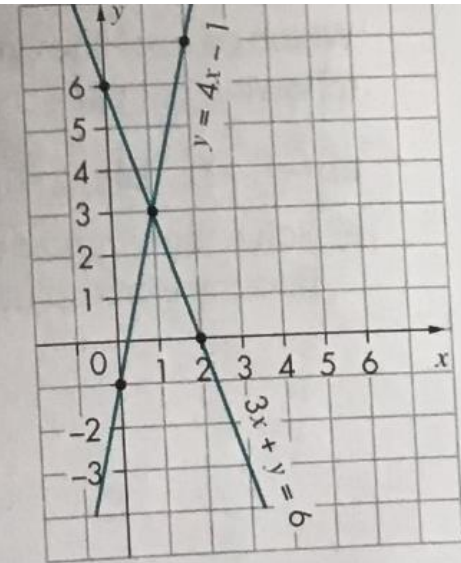
By drawing their graphs on the same grid, find the solution of these simultaneous equations.

$$3x + y = 6 \quad y = 4x - 1$$

The first graph is drawn using the cover-up method. It crosses the  $x$ -axis at  $(2, 0)$  and the  $y$ -axis at  $(0, 6)$ .

The second graph can be drawn by finding some points or by the gradient-intercept method. If you use the gradient-intercept method, you find the graph crosses the  $y$ -axis at  $-1$  and has a gradient of  $4$ .

The point where the graphs intersect is  $(1, 3)$ . So the solution to the simultaneous equations is  $x = 1, y = 3$ .



# Solving simultaneous equations graphically

## Questions

### EXERCISE 11C

#### GRADE B

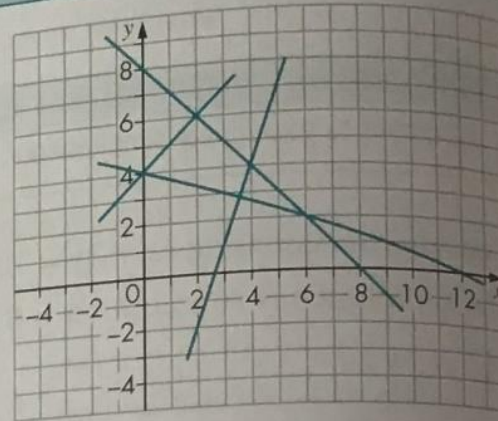
1. Use the graph to solve these simultaneous equations.

a)  $x + y = 8$   
 $y - x = 4$

b)  $x + 3y = 12$   
 $x + y = 8$

c)  $x + y = 8$   
 $3x - y = 8$

d)  $y - x = 4$   
 $x + 3y = 12$



2. Draw graphs to solve these pairs of simultaneous equations

a)  $x + 2y = 20$   
 $2x + y = 16$

b)  $x - y = 2$   
 $2x + 3y = 24$

c)  $2x - y = 12$   
 $3x + 2y = 18$

In questions 3–14, draw the graphs to find the solution of each pair of simultaneous equations.

3.  $x + 4y = 8$   
 $x - y = 3$

4.  $y = 2x - 1$   
 $3x + 2y = 12$

5.  $y = 2x + 4$   
 $y = x + 7$

6.  $y = x$   
 $x + y = 10$

7.  $y = 2x + 3$   
 $5x + y = 10$

8.  $y = 5x + 1$   
 $y = 2x + 10$

9.  $y = x + 8$   
 $x + y = 4$

10.  $y - 3x = 9$   
 $y = x - 3$

11.  $y = -x$   
 $y = 4x - 5$

12.  $3x + 2y = 18$   
 $y = 3x$

13.  $y = 3x + 2$   
 $y + x = 10$

14.  $y = \frac{x}{3} + 1$   
 $x + y = 11$

# Answers

## 11.1 Simultaneous linear equations

### Exercise 11A

- a)  $x=5, y=10$       b)  $x=18, y=6$   
c)  $x=12, y=48$       d)  $x=6, y=18$   
e)  $x=12.5, y=2.5$       f)  $x=0.5, y=4.5$
- a)  $x=13, y=7$       b)  $x=9, y=14$   
c)  $x=10, y=-4$       d)  $x=0.5, y=4$   
e)  $x=5.5, y=14.5$       f)  $x=2, y=8$
- a)  $x=4, y=1$       b)  $x=1, y=4$   
c)  $x=3, y=1$       d)  $x=2.25, y=6.5$   
e)  $x=4, y=3$       f)  $x=5, y=3$
- a)  $x=2, y=-3$       b)  $x=7, y=3$   
c)  $x=2.5, y=1.5$       d)  $x=6, y=3$   
e)  $x=0.5, y=-0.75$       f)  $x=1.5, y=0.75$
- a)  $x=5, y=1$       b)  $x=3, y=8$   
c)  $x=9, y=1$       d)  $x=3, y=-2$   
e)  $x=2, y=\frac{1}{2}$       f)  $x=-2, y=-3$   
g)  $x=-\frac{1}{2}, y=-6\frac{1}{2}$       h)  $x=3\frac{1}{2}, y=1\frac{1}{2}$   
i)  $x=-2\frac{1}{2}, y=-3\frac{1}{2}$

Area is  $6 \text{ cm}^2$ .

## 11.2 Linear and non-linear equations

### Exercise 11B

- a)  $(5, -1)$       b)  $(4, 1)$       c)  $(8, -1)$
- a)  $(1, 2)$  and  $(-2, -1)$       b)  $(-4, 1)$  and  $(-2, 2)$
- a)  $(3, 4)$  and  $(4, 3)$       b)  $(0, 3)$  and  $(-3, 0)$   
c)  $(3, 2)$  and  $(-2, 3)$
- a)  $(2, 5)$  and  $(-2, -3)$       b)  $(-1, -2)$  and  $(4, 3)$   
c)  $(3, 3)$  and  $(1, -1)$
- a)  $(-3, -3), (1, 1)$       b)  $(3, -2), (-2, 3)$   
c)  $(-2, -1), (1, 2)$       d)  $(2, -1), (3, 1)$   
e)  $(-2, 1), (3, 6)$       f)  $(1, -4), (4, 2)$   
g)  $(4, 5), (-5, -4)$

## 11.3 Using graphs to solve simultaneous linear equations

### Exercise 11C

- a)  $x=2, y=6$       b)  $x=6, y=2$       c)  $x=4, y=4$   
d)  $x=0, y=4$
- a)  $x=4, y=8$       b)  $x=6, y=4$       c)  $x=6, y=0$
- $(4, 1)$
- $(2, 3)$
- $(3, 10)$
- $5, 5$
- $(1, 5)$
- $(3, 16)$
- $(-2, 6)$
- $(-6, -9)$
- $(1, -1)$
- $(2, 6)$

# Solving simultaneous equations with 3 unknowns

Video link example, questions and solutions

[Link for example](#)

[Link for questions](#)

[Link for solutions](#)

# A Level Questions

Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

**(Total 7 marks)**

(a) By eliminating  $y$  from the equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0.$$

**(2)**

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form  $a \pm b\sqrt{3}$ , where  $a$  and  $b$  are integers.

**(5)**

**(Total 7 marks)**

# A level Questions - Solutions

$$y = 3x - 2 \quad (3x - 2)^2 - x - 6x^2 (= 0) \quad \text{M1}$$

$$9x^2 - 12x + 4 - x - 6x^2 = 0$$

$$3x^2 - 13x + 4 = 0 \text{ (or equiv., e.g. } 3x^2 = 13x - 4) \quad \text{M1 A1cso}$$

$$(3x - 1)(x - 4) = 0 \quad x = \dots \quad x = \frac{1}{3} \text{ (or exact}$$

equivalent)  $x = 4 \quad \text{M1 A1}$

$$y = -1 \quad y = 10 \quad \text{(Solutions need not be "paired")} \quad \text{M1 A1}$$

$$(a) \quad 2x^2 - x(x - 4) = 8 \quad \text{M1}$$

$$x^2 + 4x - 8 = 0 \quad (*) \quad \text{A1cso} \quad 2$$

$$(b) \quad x = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times -8)}}{2} \text{ or } (x + 2)^2 \pm 4 - 8 = 0 \quad \text{M1}$$

$$x = -2 \pm (\text{any correct expression}) \quad \text{A1}$$

$$\sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3} \text{ or } \sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3} \quad \text{B1}$$

$$y = (-2 \pm 2\sqrt{3}) - 4 \quad \text{M: Attempt at least one } y \text{ value} \quad \text{M1}$$

$$\underline{x = -2 + 2\sqrt{3}, y = -6 + 2\sqrt{3}} \quad \underline{x = -2 - 2\sqrt{3}, y = -6 - 2\sqrt{3}} \quad \text{A1} \quad 5$$