Getting ready for A Level

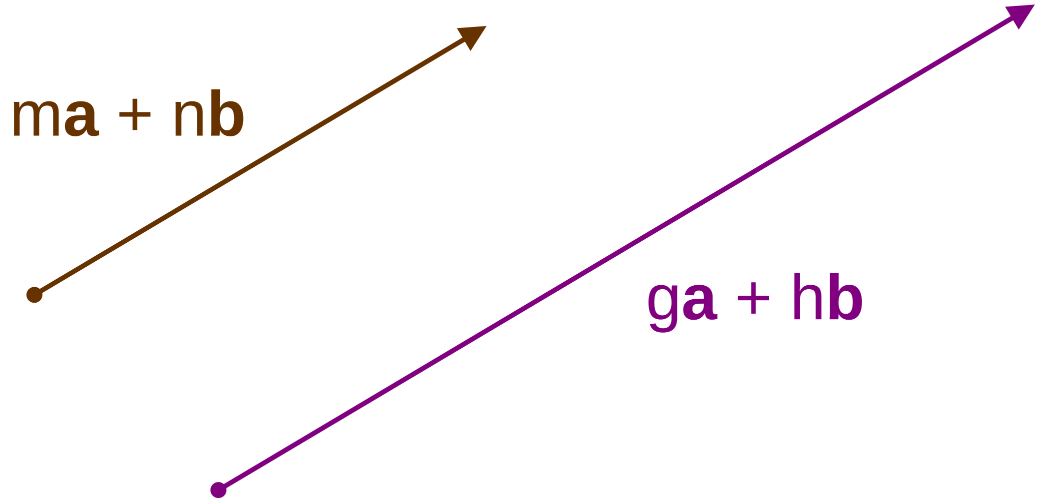
Maths

Week 7 – Vectors

Week 7 - Vectors

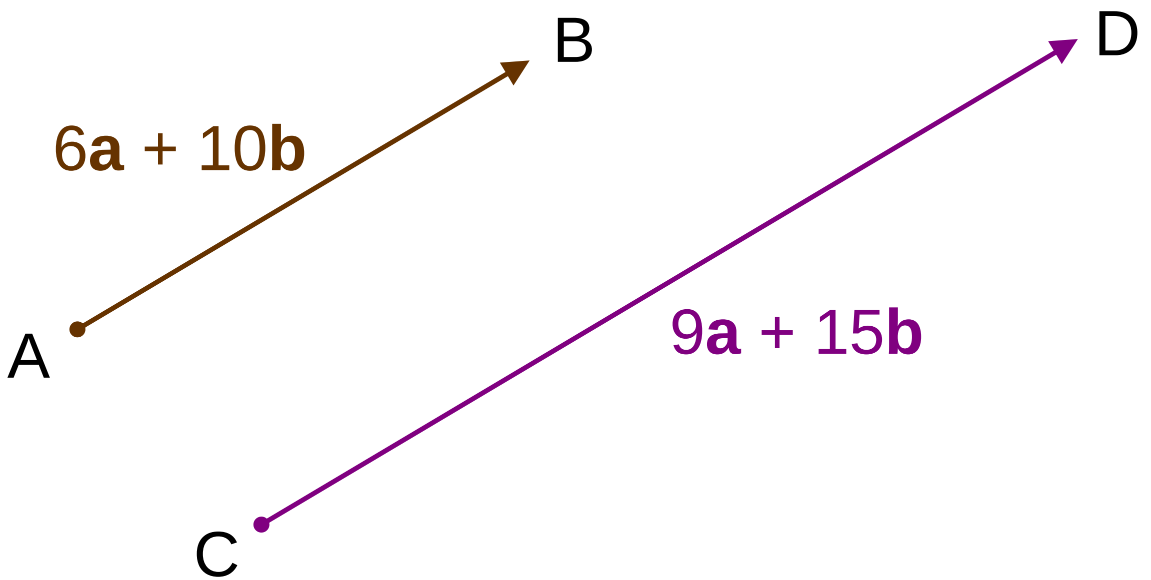
* Welcome to week 7.
* Vectors do not appear directly in A level maths but the techniques we use to solve these problems are used in mechanics and further maths A level.
* Some examples and then questions follow.

if two vectors are parallel
(or are sections of the same line)



then $\frac{g}{m} = \frac{h}{n}$

if two vectors are parallel
(or sections of the same line)



$$\frac{9}{6} = \frac{15}{10} = 1\tfrac{1}{2}$$

$$\overrightarrow{CD} = 1\tfrac{1}{2}\,\overrightarrow{AB}$$

$$\overrightarrow{AB} = \tfrac{2}{3}\,\overrightarrow{CD}$$

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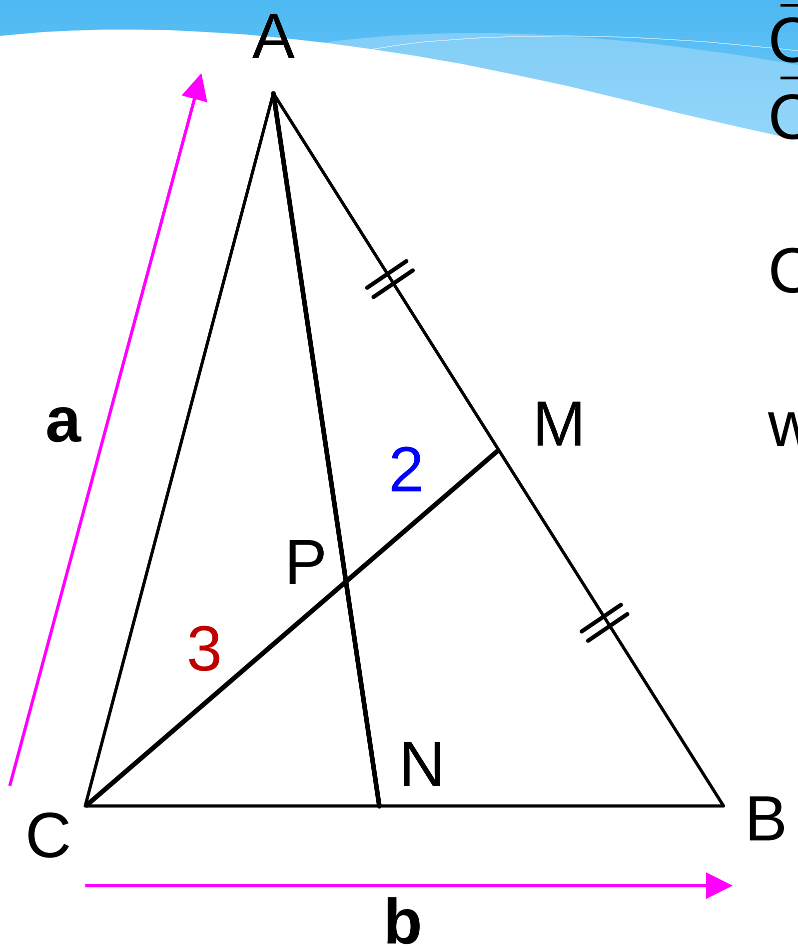


M = midpoint of AB

$\overrightarrow{CA} = \mathbf{a}$
$\overrightarrow{CB} = \mathbf{b}$

CP : PM = 3 : 2

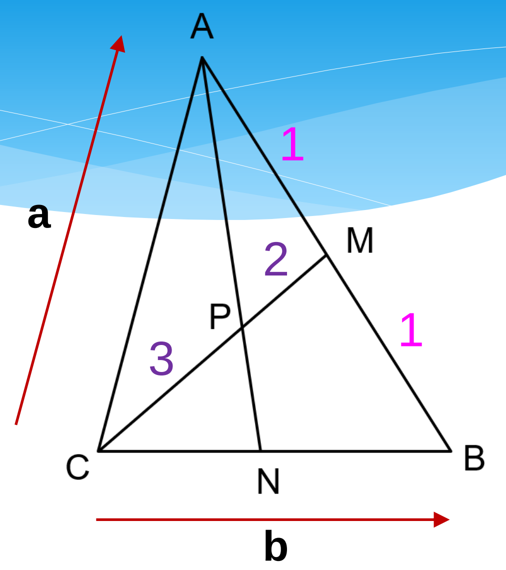
work out the ratio CN : NB

method (i)

$\overrightarrow{AB} =$

$\overrightarrow{AM} =$

$\overrightarrow{CM} =$



$\overrightarrow{CP} =$

$\overrightarrow{AP} =$

$\overrightarrow{CN} = k\,\overrightarrow{CB}$

$\overrightarrow{AN} =$

$\overrightarrow{AN}$ is a multiple of $\overrightarrow{AP}$
since in the same direction

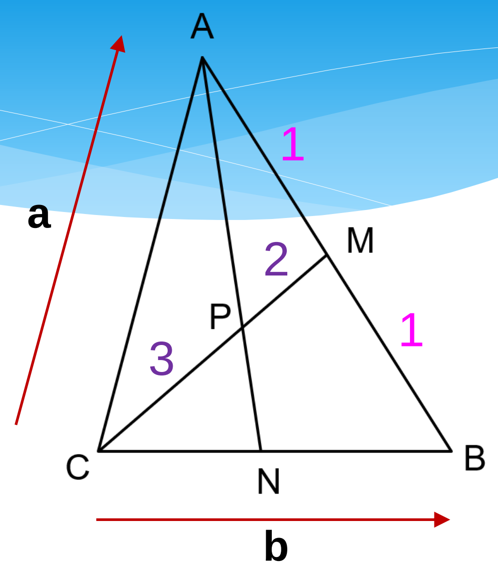
method (i)

$\overrightarrow{AB} = -\,\mathbf{a} + \mathbf{b}$

$\overrightarrow{AM} = \tfrac{1}{2}\overrightarrow{AB} = -\,\tfrac{1}{2}\,\mathbf{a} + \tfrac{1}{2}\,\mathbf{b}$

$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \mathbf{a} - \tfrac{1}{2}\,\mathbf{a} + \tfrac{1}{2}\,\mathbf{b}$

$= \tfrac{1}{2}\,\mathbf{a} + \tfrac{1}{2}\,\mathbf{b}$



$\overrightarrow{CP} = 3/5\,\overrightarrow{CM} = 3/10\,\mathbf{a} + 3/10\,\mathbf{b}$

$\overrightarrow{AP} = \overrightarrow{AC} + \overrightarrow{CP} = -\,\mathbf{a} + 3/10\,\mathbf{a} + 3/10\,\mathbf{b}$

$= -\,7/10\,\mathbf{a} + 3/10\,\mathbf{b}$

$\overrightarrow{CN} = k\,\overrightarrow{CB}$

$\overrightarrow{AN} = \overrightarrow{AC} + \overrightarrow{CN} = -\,\mathbf{a} + k\,\mathbf{b}$

$\overrightarrow{AN}$ is a multiple of $\overrightarrow{AP}$ since in the same direction

$-\,7/10 \times 10/7 = -\,1$ so $k = 3/10 \times 10/7 = 3/7$

method (ii)

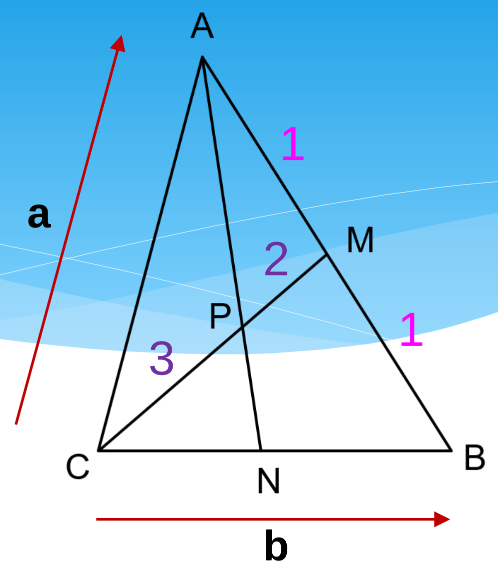
$\overrightarrow{CM}$ = ½ **a** + ½ **b**

$\overrightarrow{CP}$ = 3/5 $\overrightarrow{CM}$ = 3/10 **a** + 3/10 **b**

$\overrightarrow{AN}$ = $\overrightarrow{AC}$ + $\overrightarrow{CN}$ = – **a** + k **b**

$\overrightarrow{PN}$ = $\overrightarrow{PC}$ + $\overrightarrow{CN}$ = – 3/10 **a** – 3/10 **b** + k **b**

= – 3/10 **a** + (k – 3/10) **b**



$\overrightarrow{AN}$ is a multiple of $\overrightarrow{PN}$ since in the same direction

$$\frac{k - 3/10}{k} = \frac{-3/10}{-1}$$

7/10 k = 3/10

k = 3/7

method (iii)

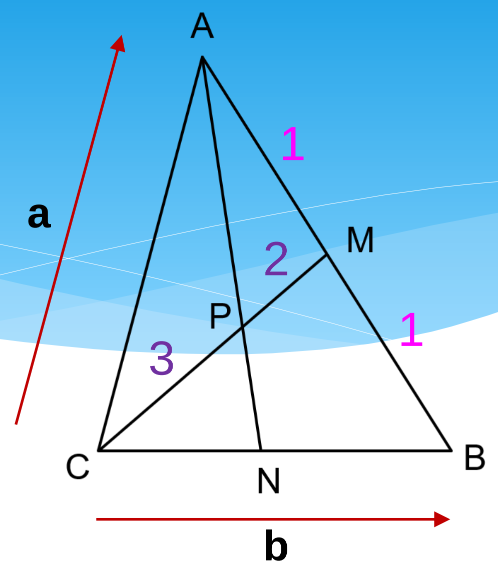
$\overrightarrow{CM} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$\overrightarrow{CP} = 3/5\ \overrightarrow{CM} = 3/10\ \mathbf{a} + 3/10\ \mathbf{b}$

$\overrightarrow{AP} = -\ 7/10\ \mathbf{a} + 3/10\ \mathbf{b}$

$\overrightarrow{AN} = f\ \overrightarrow{AP} = -\ 7/10\ f\ \mathbf{a} + 3/10\ f\ \mathbf{b}$

$\overrightarrow{CN} = \overrightarrow{CA} + \overrightarrow{AN} = \mathbf{a} - 7/10\ f\ \mathbf{a} + 3/10\ f\ \mathbf{b}$

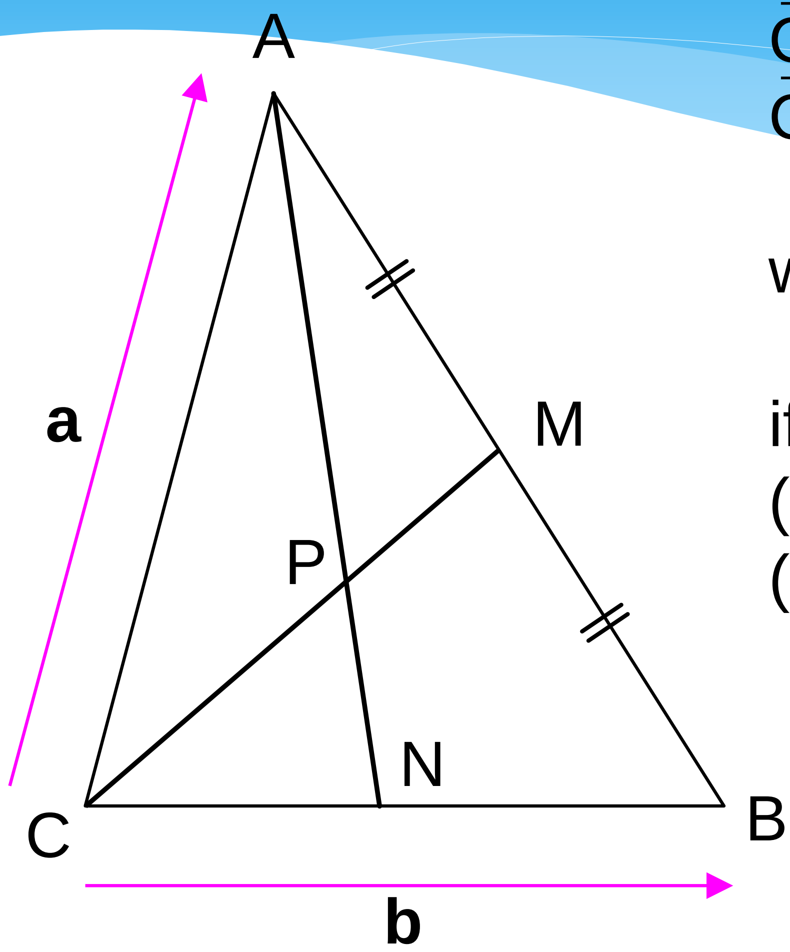


$\overrightarrow{CN}$ is a multiple of $\overrightarrow{CB}$ (= **b**)
since in the same direction
so the coefficient of **a** for $\overrightarrow{CN}$
must be zero

$1 - 7/10\ f = 0$

$f = 10/7$

$\overrightarrow{CN} = 3/7\ \mathbf{b}$



M = midpoint of AB

$\overrightarrow{CA} = \mathbf{a}$
$\overrightarrow{CB} = \mathbf{b}$

work out the ratio CN : NB

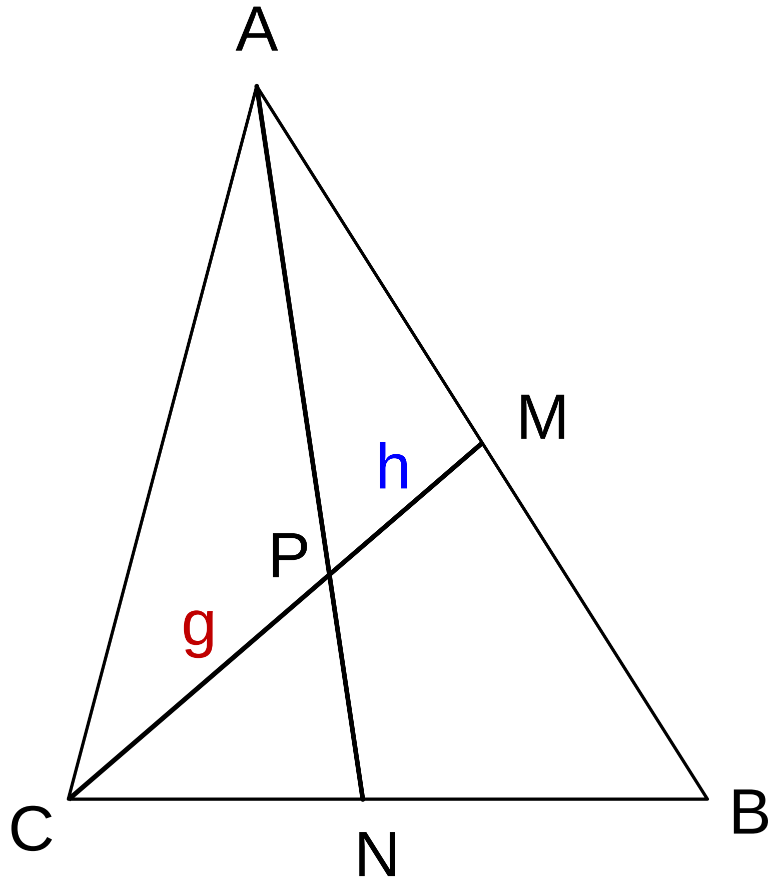
if
(i) CP : PM = 2 : 1
(ii) CP : PM = 3 : 1

(i) 1 : 1
(ii) 3 : 2

what if CP : PM = g : h ?

M is the midpoint of AB

$\overrightarrow{CA} = \mathbf{a}$
$\overrightarrow{CB} = \mathbf{b}$



CP : PM = g : h

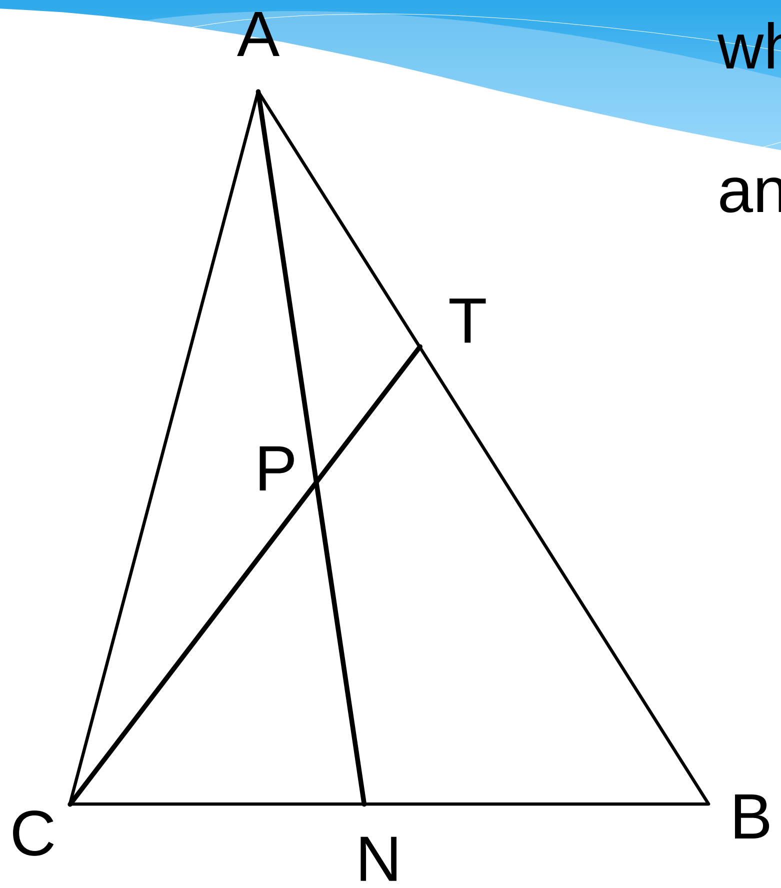
CN : NB = g : 2h

and NP : PA = g : (2h + g)

generalisation

what if AT : TB = 1 : 2 ?

and CP : PT = g : h ?



further generalisation

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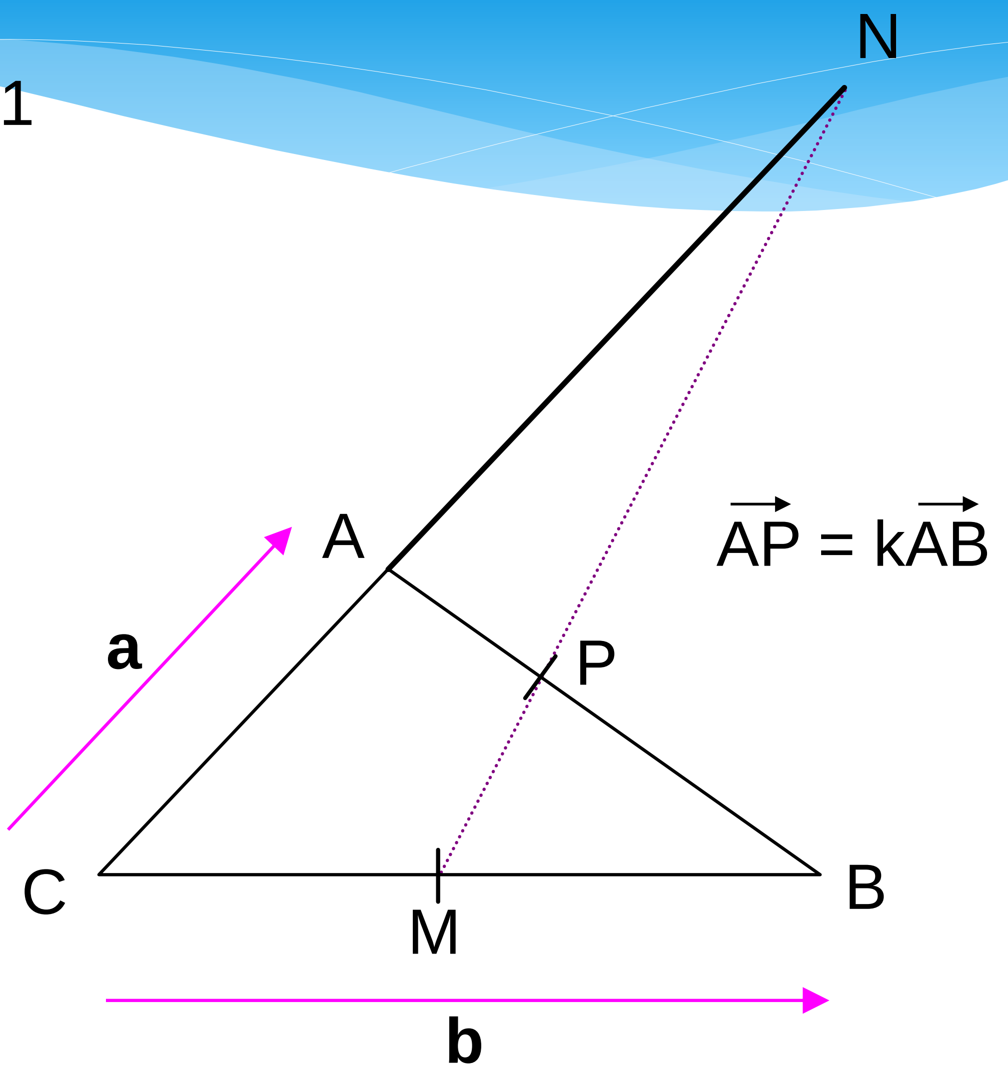
M = midpoint of CB

$\overrightarrow{CA} = \mathbf{a}$
$\overrightarrow{CB} = \mathbf{b}$

AN : CA = 2 : 1

MPN is a straight line
work out the ratio AP : PB

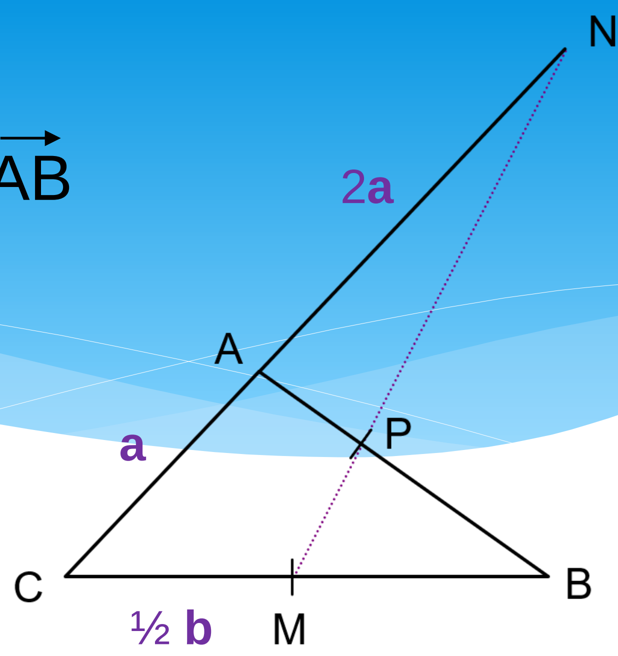


$\overrightarrow{AN} = 2\mathbf{a}$ $\overrightarrow{CM} = \frac{1}{2}\mathbf{b}$ $\overrightarrow{AP} = k\overrightarrow{AB}$

$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ $\overrightarrow{AP} = -k\mathbf{a} + k\mathbf{b}$

$\overrightarrow{NP} = \overrightarrow{NA} + \overrightarrow{AP} = -2\mathbf{a} - k\mathbf{a} + k\mathbf{b}$

$\overrightarrow{NM} = \overrightarrow{NC} + \overrightarrow{CM} = -3\mathbf{a} + \frac{1}{2}\mathbf{b}$



$\overrightarrow{NM}$ is a multiple of $\overrightarrow{NP}$
since in the same direction

$$\frac{2+k}{3} = \frac{k}{\frac{1}{2}}$$

$$2 + k = 6k$$

$$2 = 5k$$

k = 2/5

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$	1	This mark is given for finding a vector expression for $\overrightarrow{AB}$
$\overrightarrow{MN} = -\frac{1}{2}\mathbf{b} + \mathbf{a} + 2\mathbf{a}$ $= -\frac{1}{2}\mathbf{b} + 3\mathbf{a}$	1	This mark is given for finding a vector expression for $\overrightarrow{MN}$
$\overrightarrow{PN} = -k(\mathbf{b} - \mathbf{a}) + 2\mathbf{a}$ $= -k\mathbf{b} + (2 + k)\mathbf{a}$	1	This mark is given for finding a vector expression for $\overrightarrow{PN}$
Since $\overrightarrow{MN}$ is a multiple of $\overrightarrow{PN}$ $\frac{-\frac{1}{2}}{-k} = \frac{3}{(2+k)}$ $-\frac{1}{2}(2 + k) = -3k$	1	This mark is given for recognising that $\overrightarrow{MN}$ is a multiple of $\overrightarrow{PN}$ and comparing coefficients of **a** and **b**
$k = \frac{2}{5}$	1	This mark is given for the correct answer only

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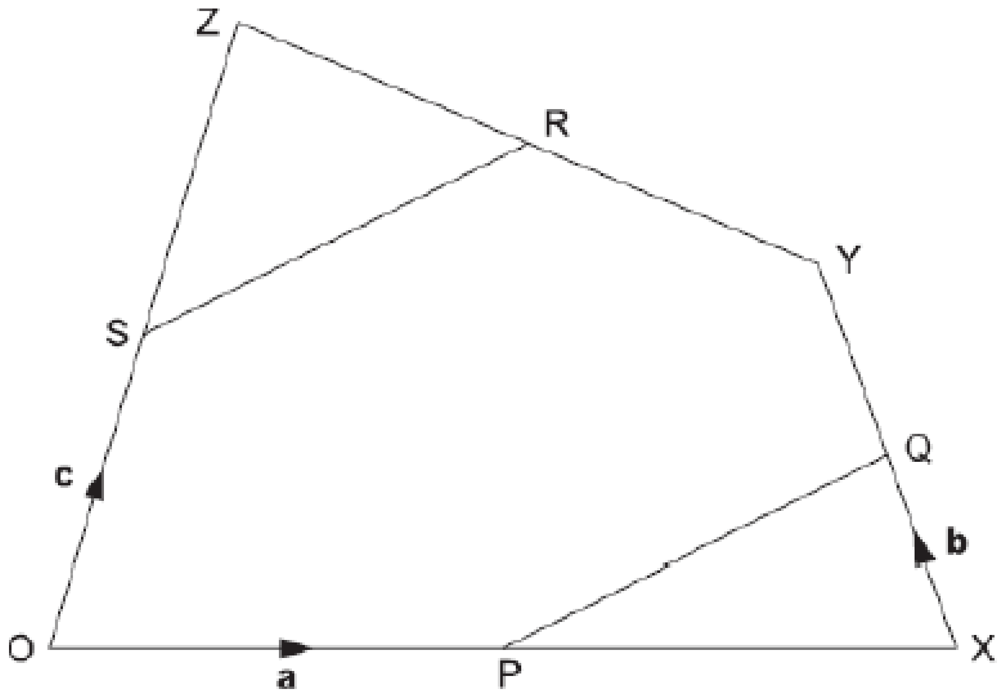
Question 1

P, Q, R and S are the midpoints of OX, XY, YZ and OZ respectively.



$\overrightarrow{OP} = \mathbf{a}$, $\overrightarrow{XQ} = \mathbf{b}$ and $\overrightarrow{OS} = \mathbf{c}$.

Show that PQ is parallel to SR.

Question 2

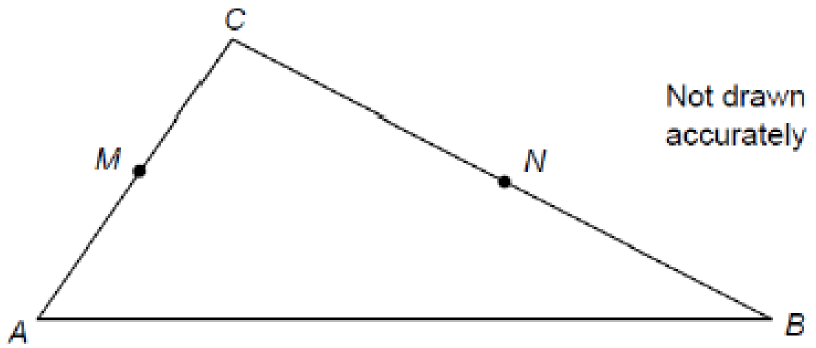
In triangle *ABC*

M is the midpoint of *AC*

N is the point on *BC* where $BN : NC = 2 : 3$

$\overrightarrow{AC} = 2\mathbf{a}$

$\overrightarrow{AB} = 3\mathbf{b}$



Not drawn accurately

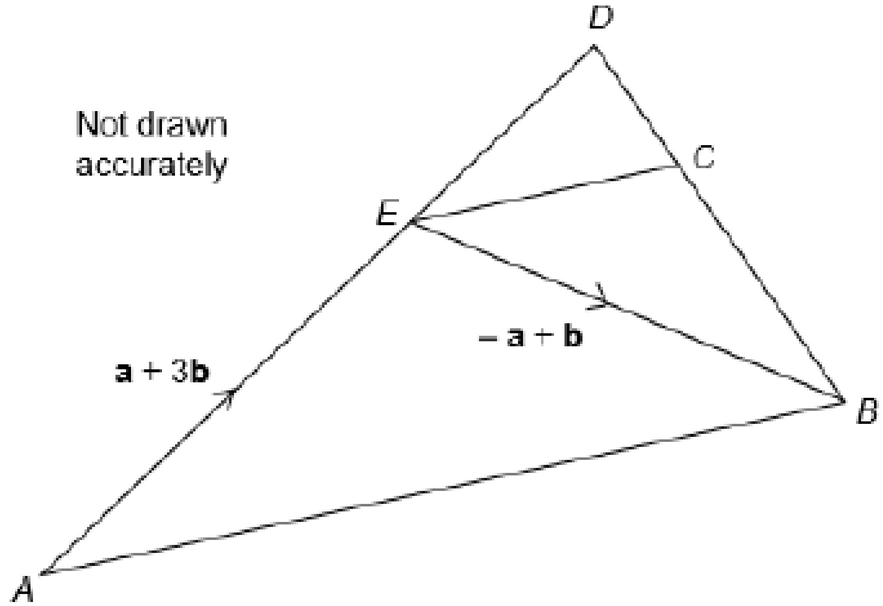
Work out $\overrightarrow{MN}$ in terms of **a** and **b**.

Give your answer in its simplest form.

Use your answer to part (a) to explain why *MN* is not parallel to *AB*.

Question 3

AED is a straight line.



$\overrightarrow{AE} = \mathbf{a} + 3\mathbf{b}$

$\overrightarrow{EB} = -\mathbf{a} + \mathbf{b}$

a) Work out the vector $\overrightarrow{AB}$

b) Also $\overrightarrow{ED} = \frac{1}{3}\overrightarrow{AE}$ and $\overrightarrow{DC} = -\frac{1}{3}\mathbf{a}$

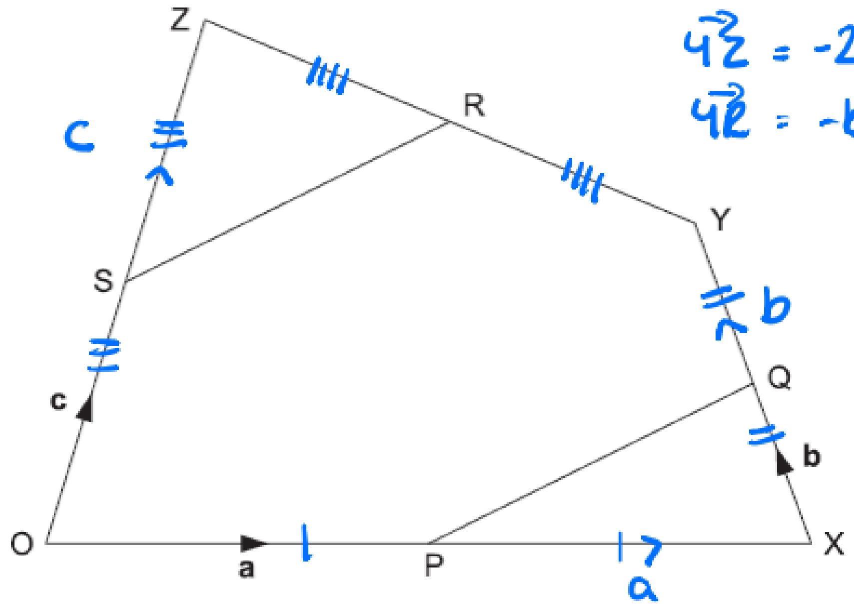
Prove that EC is parallel to AB.

Answers – Question 1

3. P, Q, R and S are the midpoints of OX, XY, YZ and OZ respectively.

$\vec{YZ} = -2b - 2a + 2c$

$\vec{YR} = -b - a + c$



$\overrightarrow{OP} = \mathbf{a}$, $\overrightarrow{XQ} = \mathbf{b}$ and $\overrightarrow{OS} = \mathbf{c}$.

Show that PQ is parallel to SR.

$\vec{PQ} = a + b$

$\vec{SR} = \vec{SZ} + \vec{ZR}$

$= c + b + a - c$

$= a + b$

$\therefore$ PQ is parallel to SR.

[5]

Answers Question 2

6. In triangle *ABC*

   M is the midpoint of *AC*

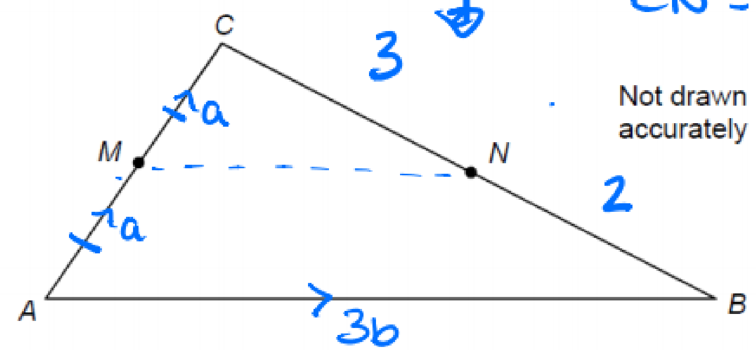
   N is the point on *BC* where *BN* : *NC* = 2 : 3

   $\overrightarrow{AC} = 2\mathbf{a}$

   $\overrightarrow{AB} = 3\mathbf{b}$

$CB = 3b - 2a$

$CN = \frac{3}{5}(3b - 2a)$



Not drawn accurately

a) Work out $\overrightarrow{MN}$ in terms of **a** and **b**.

   Give your answer in its simplest form.

$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CN} = a + \frac{3}{5}(3b - 2a) = a + \frac{9}{5}b - \frac{6}{5}a$

$= \frac{9}{5}b - \frac{1}{5}a = \frac{1}{5}(9b - a)$

[3]

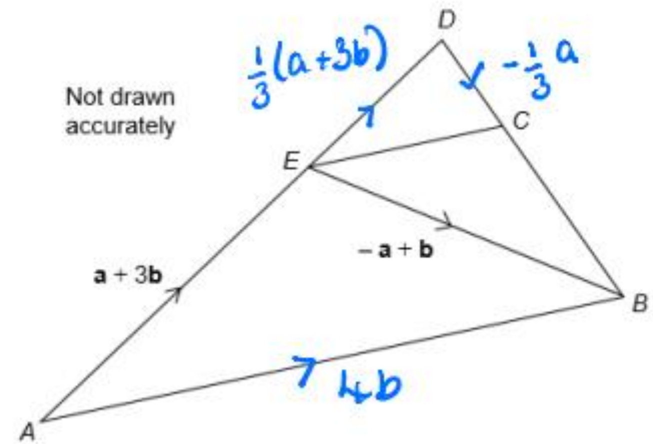
b) Use your answer to part (a) to explain why *MN* is not parallel to *AB*.

$\overrightarrow{MN}$ is not a scalar multiple of $\overrightarrow{AB}$ .. ie $\overrightarrow{MN}$ has an 'a' component $\overrightarrow{AB}$ doesn't

[1]

Answers – Question 3

10. AED is a straight line.



$\overrightarrow{AE} = \mathbf{a} + 3\mathbf{b}$

$\overrightarrow{EB} = -\mathbf{a} + \mathbf{b}$

a) Work out the vector $\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{AE} + \overrightarrow{EB} = a + 3a - a + b$

$= 4b$

b) Also $\overrightarrow{ED} = \frac{1}{3}\overrightarrow{AE}$ and $\overrightarrow{DC} = -\frac{1}{3}\mathbf{a}$

Prove that EC is parallel to AB.

$\overrightarrow{EC} = \overrightarrow{ED} + \overrightarrow{DC}$

$= \frac{1}{3}(a+3b) - \frac{1}{3}a$

$= \frac{1}{3}a + b - \frac{1}{3}a = b$

$\therefore \overrightarrow{AB} = 4\overrightarrow{EC}$

$\therefore$ AB and EC are parallel.