

Getting ready for A Level
Maths

Week 1 – Algebraic
Manipulation

Expand, Simplify and Factorise

18. Expand these expressions.

a) $4p^2(3p - q)$

b) $5t^2(2t^2 + 7)$

c) $5x(2x + 7y)$

d) $2m^2(5 - m^3)$

e) $8s^3(s + 3t)$

f) $6nm^2(m - n)$

19. Expand and simplify the following expressions.

a) $5(4x + 1) + 3(x + 2)$

b) $4(y - 2) + 5(y + 3)$

c) $2(3x - 2) - 4(x + 1)$

d) $5(2x + 3) + 6(2x - 1)$

e) $6x(2x - 3) + 2x(x + 4)$

f) $3(4x^2 - 3) + x^2(5 + 2x)$

20. Factorise the following expressions.

a) $9p^2 + 6pt$

b) $12mp - 8m^2$

c) $16a^2b + 4ab$

d) $4a^2 - 6a + 2$

e) $20xy^2 + 10x^2y + 5xy$

f) $8mt^2 - 4m^2t$

Expand, Simplify and Factorise

7. Expand and simplify.

a) $t(3t + 4) + 3t(3 + 2t)$

c) $4e(3e - 5) - 2e(e - 7)$

b) $2y(3 + 4y) + y(5y - 1)$

d) $3k(2k + p) - 2k(3p - 4k)$

8. Expand and simplify.

a) $4a(2b + 3c) + 3b(3a + 2c)$

c) $5m(2n - 3p) - 2n(3p - 2m)$

b) $3y(4w + 2t) + 2w(3y - 4t)$

d) $2r(3r + r^2) - 3r^2(4 - 2r)$

HINTS AND TIPS

Be careful with minus signs. For example, $-2(5e - 4) = -10e + 8$

Expand, Simplify and Factorise

9. A two-carriage train has f first-class seats and $2s$ standard-class seats. A three-carriage train has $2f$ first-class seats and $3s$ standard-class seats. On a weekday, 5 two-carriage trains and 2 three-carriage trains travel from Hull to Liverpool.

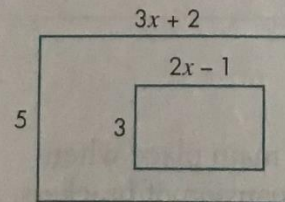
- a) Write down an expression for the total number of first-class and standard-class seats available during the day.
- b) On average in any day, half of the first-class seats are used at a cost of £60. On average in any day, three-quarters of the standard-class seats are used at a cost of £40. How much money does the rail company earn in an average day on this route? Give your answer in terms of f and s .
- c) $f=15$ and $s=80$. It costs the rail company £30 000 per day to operate this route. How much profit do they make on an average day?

10. Fill in whole-number values so that this expansion is true.

$$3(\dots x + \dots y) + 2(\dots x + \dots y) = 11x + 17y$$

11. A rectangle with sides 5 and $3x + 2$ has a smaller rectangle with sides 3 and $2x - 1$ cut from it.

Work out the remaining area.



HINTS AND TIPS

There is more than one answer. You don't have to give them all.

HINTS AND TIPS

Write out the expression for the difference between the two rectangles and then work it out.

Try to spot the pattern in each of the expressions in questions 1–10 so that you can immediately write down the expansion.

1. $(2x + 1)(2x - 1)$
2. $(5y + 3)(5y - 3)$
3. $(4m + 3)(4m - 3)$
4. $(4h - 1)(4h + 1)$
5. $(2 + 3x)(2 - 3x)$
6. $(6 - 5y)(6 + 5y)$
7. $(a + b)(a - b)$
8. $(2m - 3p)(2m + 3p)$
9. $(ab + cd)(ab - cd)$
10. $(a^2 + b^2)(a^2 - b^2)$

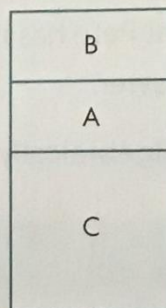
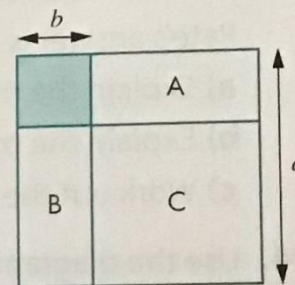
11. Imagine a square of side a units with a square of side b units cut from one corner.

a) What is the area remaining after the small square is cut away?

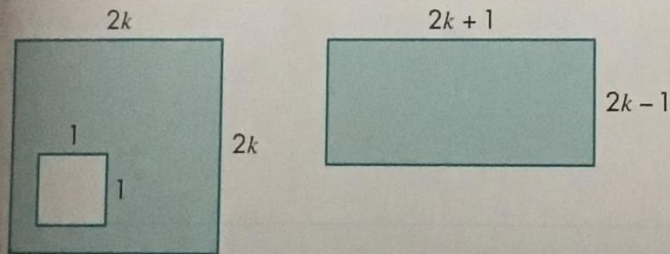
b) The remaining area is cut into rectangles, A, B and C, and rearranged as shown.

Write down the dimensions and area of the rectangle formed by A, B and C.

c) Explain why $a^2 - b^2 = (a + b)(a - b)$.



12. Explain why the areas of the shaded regions are the same.



1. Expand and simplify:

a) $(x^2 + 3x - 1)(x + 1)$

b) $(2x^2 + 5x + 1)(4x + 1)$

c) $(x^3 - 6x^2 + 2x - 3)(2x + 3)$

d) $x^2(x^4 + x^3 - 6x^2 + 2x - 3) + x(2x^2 + 3x - 1)$

GRADE A*

2. Expand and simplify:

a) $(x + 1)^3$

b) $(2x - 1)^3$

c) $(3x + 2)^3$

d) $(4x - 3)^3$

GRADE A**

3. Expand and simplify:

a) $2x^{\frac{1}{2}}(x^{\frac{1}{2}} - x^{-\frac{1}{2}})$

b) $x^{\frac{1}{3}}(x^{\frac{2}{3}} - x^{-\frac{1}{3}})$

c) $(3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}})(2x^{\frac{1}{2}} - 3x^{-\frac{1}{2}})$

d) $(4x^{\frac{1}{2}} - x^{-\frac{1}{3}})(4x^{\frac{1}{2}} + x^{-\frac{1}{3}})$

Algebraic Fractions

1.

$$\text{Solve } \frac{6x-1}{4} - \frac{5-2x}{2} = 1$$

Show clear algebraic working.

2.

$$\text{Solve } \frac{2x-1}{4} + \frac{x-1}{5} = 2$$

3.

$$\text{Solve } \frac{x-1}{2} + \frac{2x+3}{4} = 1$$

4.

$$\text{Solve } \frac{2}{5x-2} = \frac{3}{6x+1}$$

Show clear algebraic working.

5.

Solve

$$\frac{5x-7}{x-1} = x+1$$

6.

(a) Factorise $4x^2 - 1$

(b) Solve $\frac{4}{2x+1} + \frac{1}{4x^2-1} = 3$

Show clear algebraic working.

7.

$$\text{Solve } \frac{5}{(x+2)} + \frac{9}{(x-2)} = 2$$

Show clear algebraic working.

9.

$$\text{Solve } \frac{3}{(x+1)} + \frac{2}{(2x-3)} = 1$$

Show clear algebraic working.

8.

$$\text{Solve the equation } \frac{6}{x-2} - \frac{6}{x+1} = 1$$

Show clear algebraic working.

10.

$$\text{Solve the equation } \frac{3}{(x+2)} + \frac{4}{(x-3)} = 2$$

Show clear algebraic working.

Answers

18. a) $12p^3 - 4p^2q$ b) $10t^4 + 35t^2$ c) $10x^2 + 35xy$
d) $10m^2 - 2m^5$ e) $8s^4 + 24s^3t$ f) $6nm^3 - 6n^2m^2$
19. a) $23x + 11$ b) $9y + 7$ c) $2x - 8$
d) $22x + 9$ e) $14x^2 - 10x$ f) $2x^3 + 17x^2 - 9$
20. a) $3p(3p + 2t)$ b) $4m(3p - 2m)$ c) $4ab(4a + 1)$
d) $2(2a^2 - 3a + 1) = 2(2a - 1)(a - 1)$
e) $5xy(4y + 2x + 1)$ f) $4mt(2t - m)$

7. a) $9t^2 + 13t$ b) $13y^2 + 5y$
c) $10e^2 - 6e$ d) $14k^2 - 3kp$
8. a) $17ab + 12ac + 6bc$ b) $18wy + 6ty - 8tw$
c) $14mn - 15mp - 6np$ d) $8r^3 - 6r^2$
9. a) $5(f + 2s) + 2(2f + 3s) = 9f + 16s$
b) $£(270f + 480s)$
c) $£42\,450 - £30\,000 = £12\,450$
10. For x -coefficients, 3 and 1 or 1 and 4; for y -coefficients, 5 and 1 or 3 and 4 or 1 and 7
11. $5(3x + 2) - 3(2x - 1) = 9x + 13$

Exercise 2E

1. $4x^2 - 1$
2. $25y^2 - 9$
3. $16m^2 - 9$
4. $16h^2 - 1$
5. $4 - 9x^2$
6. $36 - 25y^2$

7. $a^2 - b^2$
8. $4m^2 - 9p^2$
9. $a^2b^2 - c^2d^2$
10. $a^4 - b^4$
11. a) $a^2 - b^2$
c) Dimensions: $a + b$ by $a - b$; Area: $a^2 - b^2$
d) Areas are the same, so $a^2 - b^2 = (a + b) \times (a - b)$
12. First shaded area is $(2k)^2 - 1^2 = 4k^2 - 1$. Second shaded area is $(2k + 1)(2k - 1) = 4k^2 - 1$

Answers part 2

Exercise 2G

1. a) $x^3 + 4x^2 + 2x - 1$ b) $8x^3 + 22x^2 + 9x + 1$
c) $2x^4 - 9x^3 - 14x^2 + 9$ d) $x^6 + x^5 - 6x^4 + 4x^3 - x$
2. a) $x^3 + 3x^2 + 3x + 1$ b) $8x^3 - 12x^2 + 6x - 1$
c) $27x^3 + 54x^2 + 36x + 8$ d) $64x^3 - 144x^2 + 108x - 27$
3. a) $2x - 2$ b) $x - 1$ c) $6x - 13 + \frac{6}{x}$ d) $16x - x^{-1}$